18-4 (a) Find the frequency at which the radiant energy per unit frequency interval of a blackbody is a maximum. How does this compare with the frequency at which the radiant energy per unit wavelength interval is a maximum?

(b) Find the frequency at which the cosmic background radiation is a maximum and compare your result with that shown in Figure 18.3. Assume that $T = 2.7\,\mathrm{K}$.

18-7 For photons, where there is no restriction on the total number of particles, the partition function is independent of N, as are the physical properties of a photon gas. The partition function z for a *single oscillator* is given by (see Section 15.2)

$$\ln z = -\ln(1 - e^{-h\nu/k_B T}),$$

ignoring the zero-point energy. The number of single particle (photon) states in a volume V in the frequency range ν to $\nu + d\nu$ is

$$g(\nu)d\nu = \frac{8\pi V}{c^3}\nu^2 d\nu.$$

Therefore, the partition function Z of the photon *gas* is the sum over states given by

$$\ln Z = -\frac{8\pi V}{c^3}\int_0^\infty \nu^2 \ln(1 - e^{-h\nu/k_B T})d\nu.$$

(a) Show that integration by parts leads to the equation

$$\ln Z = \frac{8\pi V}{3}\left(\frac{k_B T}{hc}\right)^3 \int_0^\infty \frac{x^3 dx}{e^x - 1},$$

where $x \equiv h\nu/k_B T$.

(b) Referring to Appendix D, show that

$$\ln Z = \frac{8\pi^5}{45}\left(\frac{k_B T}{hc}\right)^3 V.$$

18-14 In a Bose-Einstein condensation experiment, 10^7 rubidium-87 atoms were cooled down to a temperature of 200 nK. The atoms were confined to a volume of approximately $10^{-15}\,\mathrm{m}^3$.

(a) Calculate the Bose temperature T_B.

(b) Determine how many atoms were in the ground state at 200 nK.

(c) Calculate the ratio $k_B T/\varepsilon_0$, where T = 200 nK and where the ground state energy ε_0 is given by

$$\varepsilon_0 = \frac{3h^2}{8mV^{2/3}}.$$