Statistical Physics

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16-1 The partition function of an Einstein solid is

$$Z = \frac{e^{-\theta_E/2T}}{1 - e^{-\theta_E/T}},$$

where θ_E is the Einstein temperature. Treat the crystalline lattice as an assembly of $3N$ distinguishable oscillators.

(a) Calculate the Helmholtz function F.

(b) Calculate the entropy S.

(c) Show that the entropy approaches zero as the temperature goes to absolute zero. Show that at high temperatures, $S \approx 3Nk_B[1 + \ln(T/\theta_E)]$. Sketch $S/3Nk_B$ as a function of T/θ_E.

16-2 Show that the inclusion of the zero-point energy in Equation (16.14) gives a term $(9/8)Nk_B\theta_D$ that is added to the integral. Since the term is a constant, it does not contribute to the heat capacity.

16-4 (a) The heat capacity can be expressed in terms of the Debye function $D(\theta_D/T)$ by noting that

$$\int_0^{\theta_D/T} \frac{x^4 e^x}{(e^x - 1)^2} dx = \int_0^{\theta_D/T} x^4 d\left(\frac{-1}{e^x - 1}\right)$$

and integrating by parts. Show that

$$\frac{C_V}{3Nk_B} = 4D\left(\frac{\theta_D}{T}\right) - \frac{3(\theta_D/T)}{e^{\theta_D/T} - 1}.$$

(b) Calculate and plot $C_V/(3Nk_B)$ for $T/\theta_D = 0.2, 0.4, 0.6, 0.8,$ and 1.0. The corresponding values of the Debye funciton are:

T/θ_D	0.2	0.4	0.6	0.8	1.0
$D(\theta_D/T)$	0.117	0.354	0.496	0.608	0.674