Statistical Physics

May 8, 2025

15-2 (a) For a system of localized *distinguishable* oscillators, Boltzmann statistics applies. Show that the entropy S is given by

$$S = -k_B \sum_j N_j \ln\left(\frac{N_j}{N}\right).$$

(b) Substitute the Boltzmann distribution in the previous result to show that

$$S = \frac{U}{T} + Nk_B \ln Z.$$

(c) Using the expressions derived in the text for U and T, prove that

$$S = Nk_B\left[\frac{\theta/T}{e^{\theta/T}-1} - \ln(1-e^{-\theta/T})\right],$$

where $\theta = h\nu/k_B$. Examine the behavior of S as T approaches zero.

15-4 (a) In the low temperature approximation of Section 15-4, show that the Helmholtz function for rotation is

$$F_{\text{rot}} = -3Nk_BTe^{-2\theta_{\text{rot}}/T}.$$

(b) Use the reciprocal relation $S = -(\partial F/\partial T)_V$ to find the entropy S_{rot} in the same approximation. Note that $S \to 0$ as $T \to 0$, in agreement with the third law.

15-9 Using the relation

$$P = Nk_BT\left(\frac{\partial \ln Z}{\partial V}\right)_T,$$

show that the equation of state of a diatomic gas is the same as that of a monatomic gas.