Statistical Physics

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13-8 Show that for a system of a large number N of bosons at very low temperature (such that they are all in the nondegenerate lowest energy state $\varepsilon = 0$), the chemical potential varies with temperature according to

$$\mu \xrightarrow{T\to 0} -\frac{k_B T}{N}.$$

13-10 (a) Using results from Chapter 9, show that

$$\mu = -T\left(\frac{\partial S}{\partial N}\right)_{U,V}.$$

(b) It follows from the statistical definition of the entropy that

$$\Delta \ln \omega \approx -\frac{\mu}{k_B T}\Delta N.$$

Consider a system with a chemical potential $\mu = -0.3\,\mathrm{eV}$. By what factor is the number of possible microstates of the system increased when a single particle is added to it at room temperature?

13-12 A system with two nondegenerate energy levels ε_0 and ε_1 ($\varepsilon_1 > \varepsilon_0 > 0$) is populated by N distinguishable particles at temperature T.

(a) Show that the average energy per particle is given by

$$u \equiv \frac{U}{N} = \frac{\varepsilon_0 + \varepsilon_1 e^{-\beta\Delta\varepsilon}}{1 + e^{-\beta\Delta\varepsilon}}, \quad \Delta\varepsilon \equiv \varepsilon_1 - \varepsilon_0, \quad \beta \equiv 1/k_B T.$$

(b) Show that when $T \to 0$,

$$u \approx \varepsilon_0 + \Delta\varepsilon e^{-\beta\Delta\varepsilon},$$

and when $T \to \infty$,

$$u \approx \frac{1}{2}(\varepsilon_0 + \varepsilon_1) - \frac{1}{4}\beta(\Delta\varepsilon)^2.$$

(c) Show that the specific heat at constant volume is

$$c_v = k_B\left(\frac{\Delta\varepsilon}{k_B T}\right)^2 \frac{e^{-\Delta\varepsilon/k_B T}}{(1 + e^{-\Delta\varepsilon/k_B T})^2}.$$

(d) Compute c_v in the limits $T \to 0$ and $T \to \infty$ and make a careful sketch of c_v versus $\Delta\varepsilon/k_B T$.