Statistical Physics March 27, 2025

13-1 Use the method of Lagrange multipliers to solve the following problems;

(a) A rectangle has a base x and a height y, where $x + y = 8$. Find the values of x and y that give the maximum area.

(b) Find the area of the largest rectangle that can be inscribed in the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

What percentage of the area of the ellipse does the rectangle occupy?

13-2 Five identical noninteracting particles occupy the k-th energy level, which is tenfold degenerate. How many possible microstates are there if

(a) the particles are bosons?

(b) the particles are fermions?

13-3 A model thermodynamic system in which the allowed nondegenerate states have energies 0, ε, 2ε, 3ε, ..., consisting of *four* particles with total energy $U = 6\varepsilon$. Identify the possible distributions of particles, evaluate $\Omega = \sum_k \omega_k$, and work out the average occupation numbers for the various energy levels.

(a) when the particles are gaseous bosons;

(b) when the particles are gaseous fermions.