Statistical Physics

February 27, 2025

11.1 The distribution of particle speeds of a certain hypothetical gas is given by

$$N(v)dv = Ave^{-v/v_0}dv,$$

where A and v_0 are constants.

(a) Determine A so that $f(v) \equiv N(v)/N$ is a true probability density function; i.e., $\int_0^\infty f(v)dv = 1$. Sketch $f(v)$ versus v.

(b) Find $\bar{v}$ and $v_{\rm rms}$ in terms of v_0.

(c) Differentiate $f(v)$ with respect to v and set the result equal to zero to find the most probable speed v_m.

(d) The standard deviation of the speeds from the mean is defined as

$$\sigma \equiv \left[\overline{(v-\bar{v})^2}\right]^{1/2},$$

where the bar denotes the mean value. Show that

$$\sigma = \sqrt{\overline{v^2} - (\bar{v})^2}$$

in general. What is σ for this problem?

11.3 What is the number density of molecules at a temperature of 77 K in an ultra-high vacuum at 10^{-10} torr?

11.6 At what values of the speed does the Maxwell speed distribution have half its maximum value? Give your answer as a constant time $(k_B T/m)^{1/2}$.